

Synthesis Lectures on  
Mathematics & Statistics

 SYNTHESIS  
COLLECTION OF TECHNOLOGY

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# Discrete Mathematics

A Concise Introduction

 Springer

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